COUPLING PARAMETERS OF A CONCENTRIC MULTI-ELEMENT WAVEGUIDE ARRAY

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ABSTRACT

The coupling between the rectangular waveguide applicators of a multi-element annular array looking into a layered lossy cylinder of circular cross section is analyzed theoretically. To this end, a system of coupled integral equations is derived in terms of the electric fields developed on the waveguide apertures, which is solved by expanding the unknown electric field on each aperture into waveguide normal modes and by applying a Galerkin's procedure. The self reflection coefficient and the mutual coupling coefficients are then determined and numerical results for a thirty (30) element waveguide array are computed and presented.

INTRODUCTION

Multi-element waveguide systems are used in many biomedical applications, including microwave tomographic imaging and systems providing "focusing" of electromagnetic energy inside tissues. Several experimental prototypes of microwave tomographic systems have been proposed, based on the use of a large number of waveguides, acting as emitters or receicers, placed at the periphery of a cylindrical microwave chamber [1]. Furthermore, annular multi-applicator systems have been employed to achieve "focusing" inside biological tissues, for hyperthermia treatment. To this end, mainly, the low microwave spectrum (100-1000 MHz) has been employed and continuous wave concepts have been applied [2]-[4], while recently an alternative short technique, using a multi-element waveguide array and pulsed signals of short pulse width, with a high frequency (9.5 GHz) carrier, has been proposed [5].

In a multi-element array, the influence on the radiation of each element, resulting from the other elements of the system is a significant reason for non-predictable behaviour of this type of systems. For some configurations the waves coupled to an individual element from the other radiating elements may be strong and add vectorially, producing a wave travelling towards the generator of this element that appears to be a large reflection. The practical use of waveguide multi-applicator systems in hyperthermia and in microwave tomography and the fact that the exact mechanism of interaction between system elements is important, when designing and operating a multi-port microwave device, such as this type of systems, were the main reasons for treating the associated boundary value problem theoretically.

In this paper, coupling phenomena occurring in a system, consisting of an arbitrary number (N) of identical rectangular aperture waveguide applicators, arranged around the periphery of a three-layer cylindrical lossy model of circular cross section, are studied, by using the scattering matrix notation. The excitation of each applicator can be controlled independently and the N-element system can be described as a N-

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port electromagnetic device. Therefore, the scattering matrix is a N×N matrix with complex elements S_{ij} (i=1,…,N; j=1,…,N), which will be referred to as the $\underline{S}$ matrix. The scattering parameters of a thirty-element system, using waveguide applicators of 2×1 cm^2 aperture size are computed and presented, at the operation range of the system.

MATHEMATICAL ANALYSIS

The geometry of the radiating system looking into a three-layer cylindrical lossy model is shown in Fig.1.

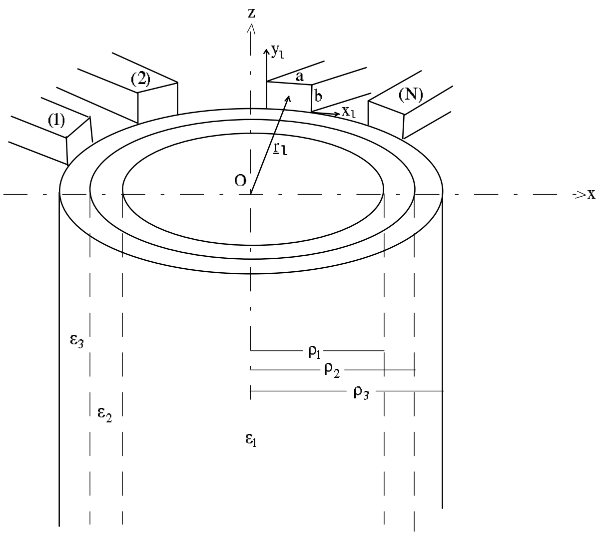

Fig.1: Multi-element waveguide array looking into a three-layer lossy medium.

The three layers are used to simulate different biological tissue media, while the external layer may be alternatively used to simulate a lossless dielectric medium which is commonly used in hyperthermia treatment as well as in microwave tomography systems. The electromagnetic properties of the layers are denoted with the corresponding relative complex permittivities $\varepsilon_1, \varepsilon_2, \varepsilon_3$. The free-space wavenumber is $k_0 = \omega\sqrt{\varepsilon_0 \mu_0}$, where ε_0 and μ_0 are the free-space permittivity and permeability, respectively. The applicators have an aperture size of a×b (b<a) and are separated by perfectly conducting flanges. It is assumed that apertures are placed at the periphery of the lossy model with the large dimension at the transverse direction circulating around the cylindrical surface and the small dimension parallel to the axis of the cylindrical model.

Cylindrical wave functions $\underline{M}^{(q)}_{m,k}(\underline{r}, k_i)$, $\underline{N}^{(q)}_{m,k}(\underline{r}, k_i)$, q=1,2 [6] are used to express the solution of the wave equation inside the tissue layers

$$\underline{E}_i(\underline{r}) = \int_{-\infty}^{+\infty} dk \sum_{m=-\infty}^{m=+\infty} \Big(a_{im}\, \underline{M}^{(1)}_{m,k}(\underline{r},k_i) + b_{im}\, \underline{N}^{(1)}_{m,k}(\underline{r},k_i) + a'_{im}\, \underline{M}^{(2)}_{m,k}(\underline{r},k_i) + b'_{im}\, \underline{N}^{(2)}_{m,k}(\underline{r},k_i) \Big) \qquad (1)$$

where i = 1,2,3 corresponds to the three regions, $k_i = k_0\sqrt{\varepsilon_i}$ and $a_{im}, b_{im}, a'_{im}, b'_{im}$ are uknown coefficients to be determined.

Then, the fields inside each waveguide are described as the superposition of the incident TE_{10} mode and an infinite number of all the reflected normal modes. Following the notation of [6], the transverse electric field inside the ℓth waveguide applicator (ℓ=1,...,N) can be written, with respect to the local cartesian coordinates system x_ℓ, y_ℓ, z_ℓ (see Fig.1) as,

$$\underline{E}^{w}_{\ell,t}(x_\ell, y_\ell, z_\ell) = A_{\ell,1}\, \underline{e}^{TE}_{1,t}(x_\ell, y_\ell) \frac{j\omega\mu_0\mu_w}{u_1} e^{-j\gamma_1 z_\ell} + \sum_{n=1}^{\infty} \Bigg(A'_{\ell,n}\, \underline{e}^{TE}_{n,t}(x_\ell, y_\ell) \frac{j\omega\mu_0\mu_w}{u_n} e^{j\gamma_n z_\ell} + B'_{\ell,n}\, \underline{e}^{TM}_{n,t}(x_\ell, y_\ell) \left(-\frac{j\lambda_n}{v_n} \right) e^{j\lambda_n z_\ell} \Bigg) \qquad (2)$$

where the subscript t is used to denote the transverse field components, $\underline{e}^{TE}_{n,t}$, $\underline{e}^{TM}_{n,t}$ are TE and TM modal fields [6] and γ_n, λ_n are the corresponding propagation constants of these fields.

By satisfying the continuity of the tangential electric and magnetic field components on the $\rho = \rho_1$ and $\rho = \rho_2$ interfaces and on the $\rho = \rho_3$ contact surface between cylindrical lossy model and radiating apertures, the following system of N coupled integral equations is obtained, in terms of an unknown transverse electric field $\underline{E}_a$ on the waveguide apertures

$$\sum_{q=1}^{N} \iint_{\Gamma_q} dx'dy' \overline{\underline{K}}_{\ell q}(x,y/x',y')\,\underline{E}_a(x',y')$$
$$= 2A_{\ell 1}\,\underline{h}_{1,t}^{TE}\left(\frac{j\gamma_1}{u_1}\right) \quad \ell = 1,\ldots,N \,/\, q = 1,\ldots,N \qquad (3)$$

where $\underline{h}_{1,t}^{TE}$ is the incident TE_{10} mode transverse magnetic field on the aperture of the ℓth waveguide, and the kernel matrices $\overline{\underline{K}}_{\ell q}(x,y/x',y')$, $q = 1,..,N \,/\, \ell = 1,\ldots,N$ indicate the effect of coupling from the qth aperture $(x',y') \in \Gamma_q$ to the ℓth aperture $(x,y) \in \Gamma_\ell$.

In order to determine the electric field distribution on the waveguide apertures and then the scattering parameters of the system (i.e. the self reflected TE_{10} mode amplitude on the excited aperture, and the coupled TE_{10} amplitudes to the other apertures), the system of integral equations (3) was solved. To this end, a Galerkin's technique was adopted, by expanding the unknown transverse electric field on each aperture $\underline{E}_{q,a}$ into wavequide normal modes,

$$\underline{E}_{q,a} = \sum_{n=1}^{\infty}\left(g_{q,n}\,\underline{e}_{n,t}^{TE} + f_{q,n}\,\underline{e}_{n,t}^{TM}\right) \quad q = 1,2,\ldots,N \qquad (4)$$

By substituting (4) into the system of coupled integral equations (3), and making use of the waveguide modes orthogonality [6], the system of integral equations (3) is converted into an infinite system of linear equations. By solving this system of linear equations, the expansion coefficients $g_{q,n}$ and $f_{q,n}$ of the field on each waveguide aperture (q=1,…,N) are computed and then the scattering matrix coefficients can be easily obtained.

NUMERICAL RESULTS AND DISCUSSION

The method developed here has been applied to study the performance of a system, using thirty (30) waveguide applicators of 2×1 cm^2 aperture size, operating at 9.5 GHz. The aplicators are considered to be placed symmetrically around the periphery of a three-layer (lossless dielectric, bone and brain tissues) lossy model, 20 cm in diameter, with thicknesses of the external lossless dielectric and bone layers $\rho_3 - \rho_2 = 2.0\,cm$, $\rho_2 - \rho_1 = 0.5\,cm$ respectively. The coupling coefficients between elements may be readily computed by exciting one element and computing the amplitude and phase of the TE_{10} mode coefficients coupled to the other waveguide applicators (mutual coupling coefficients) and the coefficient of the reflected TE_{10} mode on the same aperture (self reflection coefficient). The ratio of the induced TE_{10} mode coefficient at ℓth element (ℓ=1,…,30) to the excitation TE_{10} coefficient at qth element (q=1,..,30) gives the amplitude and the phase of the coupling coefficient $S_{\ell q}$. In order to analyze the strength of the coupling phenomena, the ratios between the mutual coupling $(S^c = S_{\ell q}, q \neq \ell)$ to self reflection $(S^r = S_{\ell q}$, $q = \ell)$ coefficients are calculated at the operation range of the system.

Numerical results for the coupling between neighbouring $(S_{ne}^c = S_{\ell q}, q = \ell + 1)$ and opposite $(S_{op}^c = S_{\ell q}, q = \ell + 15)$ applicators, are presented in Fig.2, at the operation range (1.1 f_0-1.8 f_0, f_0 being the cutoff frequency of the TE_{10} mode) of the system. In obtaining the solution, the subset of modes (TE_{10}, TE_{12}, TE_{30}, TE_{32}, TM_{12}, TM_{32})

appearing on applicator apertures has been considered to be sufficient to assure convergence and accuracy.

It is important to note that the coupling between neighbouring applicators is stronger compared to opposite applicators. At low frequencies the coupling between neighbouring applicators is of the order of -12 dB, while stronger coupling phenomena (-7 dB) are observed at the high frequency edge of the operation bandwidth (high edge=1.8 f_0, f_0 being the cutoff frequency of the TE_{10} mode).

Considering the above presented numerical results, it is clear that the magnitude of the coupling is influenced by the frequency and the distance between the elements of the array.

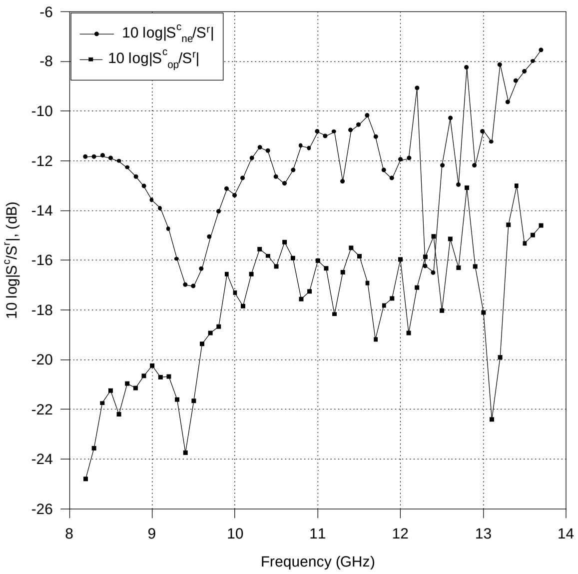

Fig.2: Coupling parameters between neighbouring and opposite applicators of a 30-element waveguide array, arranged symmetrically around the periphery of a three-layer lossy model, 20 cm in diameter.

CONCLUSION

A semianalytical solution has been presented for the power coupling between the waveguide applicators of a multi-element system radiating into a three-layer cylindrical lossy model of circular cross section. Scattering parameters indicating the effect of coupling via the radiating apertures of a thirty-element system have been computed for waveguide applicators of 2×1 cm^2 aperture size. These results are useful in designing and operating waveguide multi-applicator systems in hyperthermia and in microwave tomography.

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